CSCA Mathematics · Sample Paper

48 questions · 60 minutes · single choice, exactly one correct option · answer key and worked solutions at the end

1. Which of the following functions has the same domain and range as the function $y=10^{\lg x}$?

A. $y=x$

B. $y=\lg x$

C. $y=2^{x}$

D. $y=\frac{1}{\sqrt{x}}$

2. Given the proposition $p: \forall x \in \mathbf{R}, |x+1|>1$ and the proposition $q: \exists x>0, x^{3}=x$, then

A. both p and q are true propositions

B. both $\neg p$ and q are true propositions

C. both p and $\neg q$ are true propositions

D. both $\neg p$ and $\neg q$ are true propositions

3. Given the function $f(x)=\mathrm{e}^{-(x-1)^{2}}$. Let $a=f\left(\frac{\sqrt{2}}{2}\right), b=f\left(\frac{\sqrt{3}}{2}\right), c=f\left(\frac{\sqrt{6}}{2}\right)$, then

A. $b>c>a$

B. $b>a>c$

C. $c>b>a$

D. $c>a>b$

4. Suppose $f(x)$ is an even function with domain R and is monotonically decreasing on $(0,+\infty)$, then

A. $f\left(\log_{5} \frac{1}{4}\right)>f\left(2^{-\frac{3}{2}}\right)>f\left(2^{-\frac{2}{3}}\right)$

B. $f\left(\log_{8} \frac{1}{4}\right)>f\left(2^{-\frac{2}{3}}\right)>f\left(2^{-\frac{3}{2}}\right)$

C. $f\left(2^{-\frac{3}{2}}\right)>f\left(2^{-\frac{2}{3}}\right)>f\left(\log_{5} \frac{1}{4}\right)$

D. $f\left(2^{-\frac{2}{3}}\right)>f\left(2^{-\frac{3}{2}}\right)>f\left(\log_{5} \frac{1}{4}\right)$

5. Let the function $f(x)=\cos\left(x+\frac{\pi}{3}\right)$. Which of the following conclusions is incorrect?

A. One period of $f(x)$ is -2π

B. The graph of $y=f(x)$ is symmetric about the line $x=\frac{8\pi}{3}$

C. One zero of $f(x+\pi)$ is $x=\frac{\pi}{6}$

D. $f(x)$ is monotonically decreasing on $\left(\frac{\pi}{2}, \pi\right)$

6. Let $a=\log_{3} 2, b=\log_{5} 3, c=\frac{2}{3}$, then

A. $a<c<b$

B. $a<b<c$

C. $b<c<a$

D. $c<a<b$

7. If $a>b>0,0<c<1$, then

A. $\log_a c<\log_b c$

B. $\log_c a<\log_c b$

C. $a^c<b^c$

D. $c^a>c^b$

8. Given $a=\log_2 0.2, b=2^{0.2}, c=0.2^{0.3}$, then

A. $a<b<c$

B. $a<c<b$

C. $c<a<b$

D. $b<c<a$

9. Let the function $f(x)=\frac{1-x}{1+x}$. Which of the following functions is an odd function?

A. $f(x-1)-1$

B. $f(x-1)+1$

C. $f(x+1)-1$

D. $f(x+1)+1$

10. Let F be the focus of the parabola $C: y^2=4x$, point A lies on C, and point $B(3,0)$. If $|AF|=|BF|$, then $|AB|=$

A. 2

B. $2\sqrt{2}$

C. 3

D. $3\sqrt{2}$

11. If an asymptote of a hyperbola with center at the origin and foci on the x –axis passes through the point $(4,2)$, then its eccentricity is

A. $\sqrt{6}$

B. $\sqrt{5}$

C. $\frac{\sqrt{6}}{2}$

D. $\frac{\sqrt{5}}{2}$

12. Suppose the eccentricities of the ellipses $C_1: \frac{x^2}{a^2}+y^2=1(a>1), C_2: \frac{x^2}{4}+y^2=1$ are e_1, e_2 respectively. If $e_2=\sqrt{3}e_1$, then $a=$

A. $\frac{2\sqrt{3}}{3}$

B. $\sqrt{2}$

C. $\sqrt{3}$

D. $\sqrt{6}$

13. Given that A is a point on the parabola $C: y^2=2px(p>0)$, the distance from point A to the focus of C is 12, and its distance to the y–axis is 9, then $p=$

A. 2

B. 3

C. 6

D. 9

14. Suppose $\{a_n\}$ is a geometric sequence, and $a_1+a_2+a_3=1, a_2+a_3+a_4=2$, then $a_6+a_7+a_8=$

A. 12

B. 24

C. 30

D. 32

15. Let S_n denote the sum of the first n terms of the arithmetic sequence $\{a_n\}$. Given $S_5=S_{10}, a_5=1$, then $a_1=$

A. $\frac{7}{2}$

B. $\frac{7}{3}$

C. $-\frac{1}{3}$

D. $-\frac{7}{11}$

16. Let S_n denote the sum of the first n terms of an arithmetic sequence $\{a_n\}$. If $a_4+a_5=24, S_6=48$, then the common difference of $\{a_n\}$ is

A. 1

B. 2

C. 4

D. 8

17. Let the sets $A=\left\{x \mid x^2-4 \leq 0\right\}, B=\{x \mid 2x+a \leq 0\}$, and $A \cap B=\{x \mid -2 \leq x \leq 1\}$, then $a=$

A. –4

B. –2

C. 2

D. 4

18. If $\cos\left(\frac{\pi}{4}-\alpha\right)=\frac{3}{5}$, then $\sin 2\alpha=$

A. $\frac{7}{25}$

B. $\frac{1}{5}$

C. $-\frac{1}{5}$

D. $-\frac{7}{25}$

19. Given the function $f(x)=2\cos^2 x-\sin^2 x+2$, then

A. The minimum positive period of $f(x)$ is π , and the maximum value is 3

B. The minimum positive period of $f(x)$ is π , and the maximum value is 4

C. The minimum positive period of $f(x)$ is 2π , and the maximum value is 3

D. The minimum positive period of $f(x)$ is 2π , and the maximum value is 4

20. Let $f(x)$ be an odd function, and when $x \geq 0$, $f(x)=e^x-1$. Then when $x<0$, $f(x)=$

A. $e^{-x}-1$

B. $\mathrm{e}^{-x}+1$

C. $-\mathrm{e}^{-x}-1$

D. $-\mathrm{e}^{-x}+1$

21. A biology laboratory has 5 rabbits, of which only 3 have been measured for a certain indicator. If 3 rabbits are randomly selected from these 5, then the probability that exactly 2 of them have been measured for this indicator is

A. $\frac{2}{3}$
B. $\frac{3}{5}$
C. $\frac{2}{5}$
D. $\frac{1}{5}$

22. Given the sets $A=\{-2,0,2\}, B=\left\{x \mid x^2-x-2=0\right\}$, then $A \cap B=$

A. $\varnothing$
B. $\{2\}$
C. $\{0\}$
D. $\{-2\}$

23. Given that $z=1-2i$ and $z+a\bar{z}+b=0$, where a, b are real numbers, then

A. $a=1, b=-2$
B. $a=-1, b=2$
C. $a=1, b=2$
D. $a=-1, b=-2$

24. Let F_1, F_2 be the two foci of the hyperbola $C: x^2-\frac{y^2}{3}=1$, and let O be the origin of coordinates. If the point P lies on C and $|OP|=2$, then the area of $\triangle PF_1F_2$ is

A. $\frac{7}{2}$
B. 3
C. $\frac{5}{2}$
D. 2

25. A line l passes through one vertex and one focus of an ellipse. If the distance from the center of the ellipse to l is $\frac{1}{4}$ of the length of its minor axis, then the eccentricity of the ellipse is

A. $\frac{1}{3}$
B. $\frac{1}{2}$
C. $\frac{2}{3}$
D. $\frac{3}{4}$

26. The maximum value of the distance from the point $(0,-1)$ to the line $y=k(x+1)$ is

A. 1
B. $\sqrt{2}$
C. $\sqrt{3}$
D. 2

27. Let the left and right foci of the hyperbola $C: \frac{x^2}{a^2}-\frac{y^2}{b^2}=1 \quad(a>0, b>0)$ be F_1, F_2 respectively, and let the eccentricity be $\sqrt{5}. P$ is a point on C , and $F_1P \perp F_2P$. If the area of $\triangle PF_1F_2$ is 4 , then $a=$

A. 1
B. 2
C. 4
D. 8

28. Suppose the left and right foci of the ellipse C: $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1(a>b>0)$ are F_1, F_2, P is a point on C such that $PF_2 \perp F_1F_2, \angle PF_1F_2=30^\circ$, then the eccentricity of C is

A. $\frac{\sqrt{6}}{6}$

B. $\frac{1}{3}$

C. $\frac{1}{2}$

D. $\frac{\sqrt{3}}{3}$

29. Given that the equation $\frac{x^2}{m^2+n}-\frac{y^2}{3m^2-n}=1$ represents a hyperbola, and the distance between the two foci of this hyperbola is 4, then the range of n is

A. $(-1,3)$

B. $(-1,\sqrt{3})$

C. $(0,3)$

D. $(0,\sqrt{3})$

30. The area of obtuse triangle ABC is $\frac{1}{2}, AB=1, BC=\sqrt{2}$, then $AC=$

A. 5

B. $\sqrt{5}$

C. 2

D. 1

31. In $\triangle ABC$, $B=\frac{\pi}{4}, BC$ has a height equal to $\frac{1}{3}BC$, then $\sin A=$

A. $\frac{3}{10}$

B. $\frac{\sqrt{10}}{10}$

C. $\frac{\sqrt{5}}{5}$

D. $\frac{3\sqrt{10}}{10}$

32. The graph of the function $f(x)=\sin\left(\omega x+\frac{\pi}{3}\right)(\omega>0)$ is shifted to the left by $\frac{\pi}{2}$ units to obtain the curve C. If C is symmetric about the y–axis, then the minimum value of ω is

A. $\frac{1}{6}$

B. $\frac{1}{4}$

C. $\frac{1}{3}$

D. $\frac{1}{2}$

33. The maximum value of the function $f(x)=\frac{1}{5}\sin\left(x+\frac{\pi}{3}\right)+\cos\left(x-\frac{\pi}{6}\right)$ is

A. $\frac{6}{5}$

B. 1

C. $\frac{3}{5}$

D. $\frac{1}{5}$

34. Given that the function $f(x)=\sin(\omega x+\varphi)$ is monotonically increasing on the interval $\left(\frac{\pi}{6},\frac{2\pi}{3}\right)$, and the lines $x=\frac{\pi}{6}$ and $x=\frac{2\pi}{3}$ are two axes of symmetry of the graph of the function $y=f(x)$, then $f\left(-\frac{5\pi}{12}\right)=$

A. $-\frac{\sqrt{3}}{2}$

B. $-\frac{1}{2}$

C. $\frac{1}{2}$

D. $\frac{\sqrt{3}}{2}$

35. Given that all terms of the geometric sequence $\{a_n\}$ are positive, the sum of its first 4 terms is 15, and $a_5 = 3a_3 + 4a_1$, then $a_3 =$

- **A.** 16
- **B.** 8
- **C.** 4
- **D.** 2

36. Let S_n denote the sum of the first n terms of a geometric sequence $\{a_n\}$. If $S_2 = 4, S_4 = 6$, then $S_6 =$

- **A.** 7
- **B.** 8
- **C.** 9
- **D.** 10

37. In $\triangle ABC$, the sides opposite to the interior angles A, B, C are a, b, c respectively. Given $b = 2, B = \frac{\pi}{6}, C = \frac{\pi}{4}$, then the area of $\triangle ABC$ is

- **A.** $2\sqrt{3}+2$
- **B.** $\sqrt{3}+1$
- **C.** $2\sqrt{3}-2$
- **D.** $\sqrt{3}-1$

38. Given the ellipse $\frac{x^2}{9}+\frac{y^2}{6}=1, F_1, F_2$ are its two foci, O is the origin, P is a point on the ellipse, and $\cos\angle F_1PF_2 = \frac{3}{5}$, then $|PO| =$

- **A.** $\frac{2}{5}$
- **B.** $\frac{\sqrt{30}}{2}$
- **C.** $\frac{3}{5}$
- **D.** $\frac{\sqrt{35}}{2}$

39. Given that one asymptote of the hyperbola $C: \frac{x^2}{a^2}-\frac{y^2}{b^2}=1 \quad (a>0, b>0)$ has equation $y=\frac{\sqrt{5}}{2}x$, and the hyperbola has common foci with the ellipse $\frac{x^2}{12}+\frac{y^2}{3}=1$, then the equation of C is

- **A.** $\frac{x^2}{8}-\frac{y^2}{10}=1$
- **B.** $\frac{x^2}{4}-\frac{y^2}{5}=1$
- **C.** $\frac{x^2}{5}-\frac{y^2}{4}=1$
- **D.** $\frac{x^2}{4}-\frac{y^2}{3}=1$

40. If a circle passing through the point $(2,1)$ is tangent to both coordinate axes, then the distance from the center of the circle to the line $2x-y-3=0$ is

- **A.** $\frac{\sqrt{5}}{5}$
- **B.** $\frac{2\sqrt{5}}{5}$
- **C.** $\frac{3\sqrt{5}}{5}$
- **D.** $\frac{4\sqrt{5}}{5}$

41. Let S_n denote the sum of the first n terms of the arithmetic sequence $\{a_n\}$. Given that $S_4 = 0, a_5 = 5$, then

- **A.** $a_n = 2n-5$
- **B.** $a_n = 3n-10$
- **C.** $S_n = 2n^2-8n$
- **D.** $S_n = \frac{1}{2}n^2-2n$

42. Given that α is an acute angle and $\cos\alpha = \frac{1+\sqrt{5}}{4}$, then $\sin\frac{\alpha}{2} =$

A. $\frac{3-\sqrt{5}}{8}$
B. $\frac{-1+\sqrt{5}}{8}$
C. $\frac{3-\sqrt{5}}{4}$
D. $\frac{-1+\sqrt{5}}{4}$

43. An equilateral hyperbola C has its center at the origin and its foci on the x axis. C intersects the directrix of the parabola $y^2 = 16x$ at point A and point $B, |AB| = 4\sqrt{3}$; then the length of the real axis of C is

A. $\sqrt{2}$
B. $2\sqrt{2}$
C. 4
D. 8

44. In $\triangle ABC$, AD is the median on side BC , and E is the midpoint of AD , then $\overrightarrow{EB} =$

A. $\frac{3}{4}\overrightarrow{\mathrm{AB}} - \frac{1}{4}\overrightarrow{\mathrm{AC}}$
B. $\frac{1}{4}\overrightarrow{\mathrm{AB}} - \frac{3}{4}\overrightarrow{\mathrm{AC}}$
C. $\frac{3}{4}\overrightarrow{\mathrm{AB}} + \frac{1}{4}\overrightarrow{\mathrm{AC}}$
D. $\frac{1}{4}\overrightarrow{\mathrm{AB}} + \frac{3}{4}\overrightarrow{\mathrm{AC}}$

45. Let S_n denote the sum of the first n terms of the arithmetic sequence $\{a_n\}$. If $3S_3 = S_2 + S_4, a_1 = 2$, then $a_5 =$

A. –12
B. –10
C. 10
D. 12

46. If $f(x) = (x+a)\ln\frac{2x-1}{2x+1}$ is an even function, then $a =$

A. –1
B. 0
C. $\frac{1}{2}$
D. 1

47. Which of the following functions has a minimum value of 4?

A. $y = x^2 + 2x + 4$
B. $y = |\sin x| + \frac{4}{|\sin x|}$
C. $y = 2^x + 2^{2-x}$
D. $y = \ln x + \frac{4}{\ln x}$

48. Suppose the function $f(x) = 2^{x(x-a)}$ is monotonically decreasing on the interval $(0,1)$, then the range of a is

A. $(-\infty, -2]$
B. $[-2, 0)$
C. $(0, 2]$
D. $[2, +\infty)$

Answer key

1 D	2 B	3 A	4 C	5 D	6 A	7 B	8 B
9 B	10 B	11 D	12 A	13 C	14 D	15 B	16 C
17 B	18 D	19 B	20 D	21 B	22 B	23 A	24 B
25 B	26 B	27 A	28 D	29 A	30 B	31 D	32 C
33 A	34 D	35 C	36 A	37 B	38 B	39 B	40 B
41 A	42 D	43 C	44 A	45 B	46 B	47 C	48 D

Worked solutions

1. D The domain and range of the function $\mathrm{y}=10^{\lg \mathrm{x}}$ are both $(0,+\infty)$. The function $y=x$ has domain and range both R , which does not meet the requirement; the function $y=\lg x$ has domain $(0,+\infty)$ and range R , which does not meet the requirement; the function $y=2^x$ has domain R and range $(0,+\infty)$, which does not meet the requirement; the function $\mathrm{y}=\frac{1}{\sqrt{\mathrm{x}}}$ has domain and range both $(0,+\infty)$, which meets the requirement. So the answer is D.

2. B For p, take $x=-1$; then $|x+1|=0<1$, so p is a false proposition and $\neg p$ is a true proposition. For q, take $x=1$; then $x^3=1^3=1=x$, so q is a true proposition and $\neg q$ is a false proposition. In summary, both $\neg p$ and q are true propositions. Hence the answer is B.

3. A Let $g(x)=-(x-1)^2$; then $g(x)$ opens downward with axis of symmetry $x=1$. Since $\frac{\sqrt{6}}{2}-1-\left(1-\frac{\sqrt{3}}{2}\right)=\frac{\sqrt{6}+\sqrt{3}}{2}-\frac{4}{2}$, and $(\sqrt{6}+\sqrt{3})^2-4^2=9+6\sqrt{2}-16=6\sqrt{2}-7>0$, we get $\frac{\sqrt{6}}{2}-1-\left(1-\frac{\sqrt{3}}{2}\right)=\frac{\sqrt{6}+\sqrt{3}}{2}-\frac{4}{2}>0$, that is, $\frac{\sqrt{6}}{2}-1>1-\frac{\sqrt{3}}{2}$. By the properties of quadratic functions, $g\left(\frac{\sqrt{6}}{2}\right)<g\left(\frac{\sqrt{3}}{2}\right)$. Since $\frac{\sqrt{6}}{2}-1-\left(1-\frac{\sqrt{2}}{2}\right)=\frac{\sqrt{6}+\sqrt{2}}{2}-\frac{4}{2}$, and $(\sqrt{6}+\sqrt{2})^2-4^2=8+4\sqrt{3}-16=4\sqrt{3}-8=4(\sqrt{3}-2)<0$, we get $\frac{\sqrt{6}}{2}-1<1-\frac{\sqrt{2}}{2}$, so $g\left(\frac{\sqrt{6}}{2}\right)>g\left(\frac{\sqrt{2}}{2}\right)$. In summary, $g\left(\frac{\sqrt{2}}{2}\right)<g\left(\frac{\sqrt{6}}{2}\right)<g\left(\frac{\sqrt{3}}{2}\right)$. Since $y=\mathrm{e}^x$ is an increasing function, $a<c<b$, that is, $b>c>a$. The answer is A.

4. C $\because f(x)$ is an even function on R, $\therefore f\left(\log_3\frac{1}{4}\right)=f\left(\log_3 4\right)$. $\therefore \log_3 4>1=2^0>2^{-\frac{3}{2}}$, and since $f(x)$ is monotonically decreasing on $(0,+\infty)$, $f\left(\log_3 4\right)<f\left(2^{-\frac{2}{3}}\right)<f\left(2^{-\frac{3}{2}}\right)$, $\therefore f\left(2^{-\frac{3}{2}}\right)>f\left(2^{-\frac{2}{3}}\right)>f\left(\log_3\frac{1}{4}\right)$. Hence C.

5. D A. The period of the function is $2k\pi$; when $k=-1$, the period is $T=-2\pi$, so A is correct. B. When $x=\frac{8\pi}{3}$, $\cos\left(x+\frac{\pi}{3}\right)=\cos\left(\frac{8\pi}{3}+\frac{\pi}{3}\right)=\cos\frac{9\pi}{3}=\cos 3\pi=-1$ is the minimum value, and then the graph of $y=f(x)$ is symmetric about the line $x=\frac{8\pi}{3}$, so B is correct. C. When $x=\frac{\pi}{6}$, $f\left(\frac{\pi}{6}+\pi\right)=\cos\left(\frac{\pi}{6}+\pi+\frac{\pi}{3}\right)=\cos\frac{3\pi}{2}=0$, so one zero of $f(x+\pi)$ is $x=\frac{\pi}{6}$, so C is correct. D. When $\frac{\pi}{2}<x<\pi$, $\frac{5\pi}{6}<x+\frac{\pi}{3}<\frac{4\pi}{3}$, and on this interval the function $f(x)$ is not monotonic, so D is incorrect. So the answer is D.

6. A Since $a=\frac{1}{3}\log_3 2^3<\frac{1}{3}\log_3 9=\frac{2}{3}=c, b=\frac{1}{3}\log_5 3^3>\frac{1}{3}\log_5 25=\frac{2}{3}=c$, we get $a<c<b$. The answer is A.

7. B $\because a>b>0, 0<c<1$, $\therefore \log_c a<\log_c b$, so B is correct. $\therefore$ when $a>b>1$, $0>\log_a c>\log_b c$, so A is incorrect; $a^c>b^c$, so C is incorrect; $c^a<c^b$, so D is incorrect.

8. B $a=\log_2 0.2<\log_2 1=0, \quad b=2^{0.2}>2^0=1, \quad 0<0.2^{0.3}<0.2^0=1$, so $0<c<1, a<c<b$. The answer is B.

9. B $f(x)=\frac{1-x}{1+x}=-1+\frac{2}{1+x}$ Translating the graph of $f(x)$ one unit to the right and one unit upward gives $g(x)=\frac{2}{x}$, which is an odd function. So the answer is B.

10. B From the given conditions, $F(1,0)$, so $|AF|=|BF|=2$, that is, the distance from point A to the directrix $x=-1$ is 2, so the x–coordinate of point A is $-1+2=1$. Without loss of generality, suppose point A is above the x–axis; substituting gives $A(1,2)$, so $|AB|=\sqrt{(3-1)^2+(0-2)^2}=2\sqrt{2}$. The answer is B.

11. D $\because$ The equations of the asymptotes are $y=\pm\frac{b}{a}x$, $\therefore 2=\frac{b}{a}\bullet 4, \frac{b}{a}=\frac{1}{2}$, $a=2b$, $c=\sqrt{a^2+b^2}=\frac{\sqrt{5}}{2}a$, $e=\frac{c}{a}=\frac{\sqrt{5}}{2}$, that is, its eccentricity is $\frac{\sqrt{5}}{2}$. Hence the answer is D.

12. A From $e_2=\sqrt{3}e_1$ we get $e_2^2=3e_1^2$, so $\frac{4-1}{4}=3\times\frac{a^2-1}{a^2}$. Since $a>1$, we obtain $a=\frac{2\sqrt{3}}{3}$. The answer is A.

13. C Let the focus of the parabola be F. By the definition of the parabola, $|AF|=x_A+\frac{p}{2}=12$, that is, $12=9+\frac{p}{2}$, which gives $p=6$.

14. D Let the common ratio of the geometric sequence $\{a_n\}$ be q . Then $a_1+a_2+a_3=a_1\left(1+q+q^2\right)=1$, and $a_2+a_3+a_4=a_1q+a_1q^2+a_1q^3=a_1q\left(1+q+q^2\right)=q=2$. Therefore $a_6+a_7+a_8=a_1q^5+a_1q^6+a_1q^7=a_1q^5\left(1+q+q^2\right)=q^5=32$. Hence the answer is D.

15. B From $S_{10}-S_5=a_6+a_7+a_8+a_9+a_{10}=5a_8=0$, we get $a_8=0$, so the common difference of the arithmetic sequence $\{a_n\}$ is $d=\frac{a_8-a_5}{3}=-\frac{1}{3}$, hence $a_1=a_5-4d=1-4\times\left(-\frac{1}{3}\right)=\frac{7}{3}$. The answer is B.

16. C $\because S_n$ is the sum of the first n terms of the arithmetic sequence $\{a_n\}$, and $a_4+a_5=24, S_6=48$, $\therefore$ $\begin{cases} a_1+3d+a_1+4d=24 \\ 6a_1+\frac{6\times5}{2}d=48 \end{cases}$, solving gives $\mathrm{a}_1=-2,\ \mathrm{d}=4$, $\therefore \{a_n\}$ has common difference 4 . The answer is C.

17. B Solving the quadratic inequality $x^2-4\le 0$ gives $A=\{x\mid -2\le x\le 2\}$, and solving the linear inequality $2x+a\le 0$ gives $B=\left\{x\mid x\le -\frac{a}{2}\right\}$. Since $A\cap B=\{x\mid -2\le x\le 1\}$, we have $-\frac{a}{2}=1$, which gives $a=-2$.

18. D Method 1° $\because\because \cos\left(\frac{\pi}{4}-\alpha\right)=\frac{3}{5}$, $\therefore \sin 2\alpha=\cos\left(\frac{\pi}{2}-2\alpha\right)=\cos 2\left(\frac{\pi}{4}-\alpha\right)=2\cos^2\left(\frac{\pi}{4}-\alpha\right)-1=2\times\frac{9}{25}-1=-\frac{7}{25}$. Method 2° $\because\because \cos\left(\frac{\pi}{4}-\alpha\right)=\frac{\sqrt{2}}{2}(\sin\alpha+\cos\alpha)=\frac{3}{5}$, $\therefore \frac{1}{2}(1+\sin 2\alpha)=\frac{9}{25}$, $\therefore \sin 2\alpha=2\times\frac{9}{25}-1=-\frac{7}{25}$. The answer is D.

19. B The function $f(x)=2\cos^2 x-\sin^2 x+2=2\cos^2 x-\sin^2 x+2\sin^2 x+2\cos^2 x=4\cos^2 x+\sin^2 x=3\cos^2 x+1$, $=3\cdot\frac{\cos 2x+1}{2}+1$, $=\frac{3\cos 2x}{2}+\frac{5}{2}$, so the minimum positive period of the function is π , and the maximum value of the function is $\frac{3}{2}+\frac{5}{2}=4$.

20. D $\because f(x)$ is an odd function. When $x<0$, $-x>0$, $f(-x)=\mathrm{e}^{-x}-1=-f(x)$, which gives $f(x)=-\mathrm{e}^{-x}+1$. The answer is D .

21. B Let the 3 rabbits that have been tested be a,b,c , and the remaining 2 be A,B . Then all ways of choosing 3 from these 5 are $\{a,b,c\},\{a,b,A\},\{a,b,B\},\{a,c,A\},\{a,c,B\},\{a,A,B\}$, $\{b,\mathrm{c},A\},\{b,\mathrm{c},B\},\{\mathrm{b},A,B\},\{\mathrm{c},A,B\}$, 10 in total. Among them, the selections in which exactly 2 have been tested are $\{a,b,A\},\{a,b,B\},\{a,c,A\},\{a,c,B\},\{b,\mathrm{c},A\},\{b,\mathrm{c},B\}$, 6 in total, so the probability that exactly 2 have been tested is $\frac{6}{10}=\frac{3}{5}$. The answer is B.

22. B $\because A=\{-2,0,2\}, B=\left\{x\mid x^2-x-2=0\right\}=\{-1,2\}$, $\therefore A\cap B=\{2\}$. Hence the answer is B.

23. A $\bar{z}=1+2\mathrm{i}$ $z+a\bar{z}+b=1-2\mathrm{i}+a(1+2\mathrm{i})+b=(1+a+b)+(2a-2)\mathrm{i}$ From $z+a\bar{z}+b=0$, we get $\begin{cases} 1+a+b=0 \\ 2a-2=0 \end{cases}$, that is, $\begin{cases} a=1 \\ b=-2 \end{cases}$ The answer is A.

24. B By the given information, we may take $F_1(-2,0), F_2(2,0)$, so $a=1, c=2$. Since $|OP|=1=\frac{1}{2}|F_1F_2|$, the point P lies on the circle with F_1F_2 as its diameter, that is, $\square F_1F_2P$ is a right triangle with the right angle at P , so $|PF_1|^2+|PF_2|^2=|F_1F_2|^2$, that is, $|PF_1|^2+|PF_2|^2=16$. Also $|PF_1|-|PF_2|=2a=2$, so $4=\left||PF_1|-|PF_2|\right|^2=|PF_1|^2+|PF_2|^2-2|PF_1||PF_2|=16-2|PF_1||PF_2|$, which gives $|PF_1||PF_2|=6$, so $S_{\triangle F_1F_2P}=\frac{1}{2}|PF_1||PF_2|=3$. Hence the answer is B.

25. B Let the equation of the ellipse be $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$. The line l passes through one vertex and one focus of the ellipse, so its equation is $\frac{x}{c}+\frac{y}{b}=1$. The distance from the center of the ellipse to l is $\frac{1}{4}$ of the length of the minor axis, which gives $\frac{1}{\sqrt{\frac{1}{c^2}+\frac{1}{b^2}}}=\frac{b}{2}$, that is $4=b^2\left(\frac{1}{c^2}+\frac{1}{b^2}\right)$, $\therefore \frac{b^2}{c^2}=3$, $\frac{a^2-c^2}{c^2}=3$, $\therefore \mathrm{e}=\frac{\mathrm{c}}{\mathrm{a}}=\frac{1}{2}$.

26. B From $y=k(x+1)$, the line passes through the fixed point $P(-1,0)$. Let $A(0,-1)$. When the line $y=k(x+1)$ is perpendicular to AP, the distance from point A to the line $y=k(x+1)$ is maximal, and it equals $|AP|=\sqrt{2}$. The answer is B.

27. A $\because \frac{c}{a}=\sqrt{5}, \therefore c=\sqrt{5}a$. By the definition of the hyperbola, $\|PF_1|-|PF_2\|=2a$. $S_{\triangle PF_1F_2}=\frac{1}{2}|PF_1|\cdot|PF_2|=4$, that is $|PF_1|\cdot|PF_2|=8$. $\because F_1P\perp F_2P,\quad \therefore |PF_1|^2+|PF_2|^2=(2c)^2$, $\therefore (|PF_1|-|PF_2|)^2+2|PF_1|\cdot|PF_2|=4c^2$, that is $a^2-5a^2+4=0$, solving gives $a=1$. The answer is A.

28. D Let $|\mathrm{PF}_2|=\mathrm{x}, \because \mathrm{PF}_2\perp \mathrm{F}_1\mathrm{F}_2, \angle \mathrm{PF}_1\mathrm{F}_2=30^\circ, \therefore |P_1|=2x, |F_1F_2|=\sqrt{3}x$. Also $|\mathrm{PF}_1|+|\mathrm{PF}_2|=2\mathrm{a}, |\mathrm{F}_1\mathrm{F}_2|=2\mathrm{c}, \therefore 2a=3x, 2c=\sqrt{3}x, \therefore$ C has eccentricity $\mathrm{e}=\frac{2\mathrm{c}}{2\mathrm{a}}=\frac{\sqrt{3}}{3}$. The answer is D.

29. A The distance between the two foci of the hyperbola is 4, $\quad\therefore c=2$. When the foci are on the x –axis, $4=\left(m^2+n\right)+\left(3m^2-n\right)$, solving gives $m^2=1$. Since the equation $\frac{x^2}{m^2+n}-\frac{y^2}{3m^2-n}=1$ represents a hyperbola, $\therefore \left(m^2+n\right)\left(3m^2-n\right)>0$, which gives $(n+1)(3-n)>0$, solving gives $-1<n<3$, that is, the range of n is $(-1,3)$. When the foci are on the y –axis, $-4=\left(m^2+n\right)+\left(3m^2-n\right)$, solving gives $m^2=-1$, which has no solution. The answer is A.

30. B $\because$ the area of obtuse triangle ABC is $\frac{1}{2}, AB=c=1, BC=a=\sqrt{2}$, $\therefore S=\frac{1}{2}\operatorname{acsin}B=\frac{1}{2}$, that is $\sin B=\frac{\sqrt{2}}{2}$. When B is obtuse, $\cos B=-\sqrt{1-\sin^2 B}=-\frac{\sqrt{2}}{2}$, and the law of cosines gives $AC^2=AB^2+BC^2-2AB\cdot BC\cdot\cos B=1+2+2=5$, that is $AC=\sqrt{5}$. When B is acute, $\cos B=\sqrt{1-\sin^2 B}=\frac{\sqrt{2}}{2}$, and the law of cosines gives $AC^2=AB^2+BC^2-2AB\cdot BC\cdot\cos B=1+2-2=1$, that is $AC=1$. In this case $AB^2+AC^2=BC^2$, so $\triangle ABC$ would be a right triangle, which does not meet the condition and is discarded. Hence $AC=\sqrt{5}$. The answer is B.

31. D $\because$ in $\triangle ABC$, $B=\frac{\pi}{4}, BC$ has a height equal to $\frac{1}{3}BC$, $\therefore \mathrm{AB}=\frac{\sqrt{2}}{3}\mathrm{BC}$. By the law of cosines: $AC=\sqrt{AB^2+BC^2-2\cdot AB\cdot BC\cdot\cos B}=\sqrt{\frac{2}{9}BC+BC^2-\frac{2}{3}BC^2}=\frac{\sqrt{5}}{3}BC$. Hence $\frac{1}{2}BC\cdot\frac{1}{3}BC=\frac{1}{2}AB\cdot AC\cdot\sin A=\frac{1}{2}\cdot\frac{\sqrt{2}}{3}BC\cdot\frac{\sqrt{5}}{3}BC\cdot\sin A$, $\therefore \sin A=\frac{3\sqrt{10}}{10}$. So the answer is D.

32. C From the given conditions, the curve C is $y=\sin\left[\omega\left(x+\frac{\pi}{2}\right)+\frac{\pi}{3}\right]=\sin\left(\omega x+\frac{\omega\pi}{2}+\frac{\pi}{3}\right)$. Since C is symmetric about the y–axis, $\frac{\omega\pi}{2}+\frac{\pi}{3}=\frac{\pi}{2}+k\pi, k\in\mathbf{Z}$, solving gives $\omega=\frac{1}{3}+2k, k\in\mathbf{Z}$. Since $\omega>0$, when $k=0$ the minimum value of ω is $\frac{1}{3}$. The answer is C.

33. A The function $\mathrm{f}(\mathrm{x})=\frac{1}{5}\sin\left(\mathrm{x}+\frac{\pi}{3}\right)+\cos\left(x-\frac{\pi}{6}\right)=\frac{1}{5}\sin\left(x+\frac{\pi}{3}\right)+\cos\left(-x+\frac{\pi}{6}\right)=\frac{1}{5}\sin\left(x+\frac{\pi}{3}\right)+\sin\left(x+\frac{\pi}{3}\right)$ $=\frac{6}{5}\sin\left(x+\frac{\pi}{3}\right)\leqslant\frac{6}{5}$. The answer is A.

34. D $\because f(x)=\sin(\omega x+\varphi)$ is monotonically increasing on the interval $\left(\frac{\pi}{6},\frac{2\pi}{3}\right)$, and $x=\frac{\pi}{6}$ and $x=\frac{2\pi}{3}$ are axes of symmetry of $y=f(x)$, $\therefore f\left(\frac{\pi}{6}\right)=-1, \left|\frac{\pi}{6}-\frac{2\pi}{3}\right|=\frac{T}{2}=\frac{2\pi}{2\omega}$, which gives $\omega=2$.

$$\therefore f\left(\frac{\pi}{6}\right)=\sin\left(\frac{\pi}{3}+\varphi\right)=-1,\ \text{i.e.}\ \frac{\pi}{3}+\varphi=2k\pi-\frac{\pi}{2}, \therefore \varphi=2k\pi-\frac{5\pi}{6},$$
$$\therefore f\left(-\frac{5\pi}{12}\right)=\sin\left(-\frac{5\pi}{6}+2k\pi-\frac{5\pi}{6}\right)=\sin\left(-\frac{5\pi}{3}\right)=\frac{\sqrt{3}}{2}$$

So the answer is D.

35. C Let the common ratio of the geometric sequence $\{a_n\}$ with positive terms be q, then $\begin{cases} a_1+a_1q+a_1q^2+a_1q^3=15, \\ a_1q^4=3a_1q^2+4a_1 \end{cases}$, solving gives $\begin{cases} a_1=1, \\ q=2 \end{cases}$, $\therefore a_3=a_1q^2=4$. Hence C.

36. A $\because S_n$ is the sum of the first n terms of the geometric sequence $\{a_n\}$, $\therefore S_2,\quad S_4-S_2,\quad S_6-S_4$ form a geometric sequence, $\therefore S_2=4,\quad S_4-S_2=6-4=2, \therefore S_6-S_4=1, \therefore S_6=1+S_4=1+6=7$. The answer is A.

37. B $\because b=2, B=\frac{\pi}{6}, C=\frac{\pi}{4}, \therefore$ by the law of sines $\frac{b}{\sin B}=\frac{c}{\sin C}$, we get $c=\frac{b\sin C}{\sin B}=\frac{2\times\frac{\sqrt{2}}{2}}{\frac{1}{2}}=2\sqrt{2}, A=\frac{7\pi}{12}, \therefore \sin A=\sin\left(\frac{\pi}{2}+\frac{\pi}{12}\right)=\cos\frac{\pi}{12}=\frac{\sqrt{2}+\sqrt{6}}{4}$, then $S_{\triangle ABC}=\frac{1}{2}bc\sin A=\frac{1}{2}\times 2\times 2\sqrt{2}\times\frac{\sqrt{2}+\sqrt{6}}{4}=\sqrt{3}+1$. The answer is B.

38. B Method 1: Let $\angle F_1PF_2=2\theta, 0<\theta<\frac{\pi}{2}$, so $S_{\triangle PF_1F_2}=b^2\tan\frac{\angle F_1PF_2}{2}=b^2\tan\theta$. From $\cos\angle F_1PF_2=\cos 2\theta=\frac{\cos^2\theta-\sin^2\theta}{\cos^2\theta+\sin^2\theta}=\frac{1-\tan^2\theta}{1+\tan^2\theta}=\frac{3}{5}$, we get $\tan\theta=\frac{1}{2}$. From the equation of the ellipse, $a^2=9, b^2=6, c^2=a^2-b^2=3$, so $S_{\triangle PF_1F_2}=\frac{1}{2}\times|F_1F_2|\times|y_p|=\frac{1}{2}\times 2\sqrt{3}\times|y_p|=6\times\frac{1}{2}$, which gives $y_p^2=3$, and then $x_p^2=9\times\left(1-\frac{3}{6}\right)=\frac{9}{2}$. Therefore $|OP|=\sqrt{x_p^2+y_p^2}=\sqrt{3+\frac{9}{2}}=\frac{\sqrt{30}}{2}$. The answer is B.

39. B The foci of the ellipse $\frac{x^2}{12}+\frac{y^2}{3}=1$ are $(\pm 3,0)$, so the foci of the hyperbola are $(\pm 3,0)$, which gives $c=3$. One asymptote of the hyperbola C: $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1 \quad (a>0,b>0)$ has equation $y=\frac{\sqrt{5}}{2}x$, so $\frac{b}{a}=\frac{\sqrt{5}}{2}$, that is, $\frac{c^2-a^2}{a^2}=\frac{5}{4}$, which gives $\frac{c}{a}=\frac{3}{2}$; solving gives $a=2, b=\sqrt{5}$. The required equation of the hyperbola is $\frac{x^2}{4}-\frac{y^2}{5}=1$. So the answer is B.

40. B Since the point $(2,1)$ on the circle lies in the first quadrant, if the center were not in the first quadrant, the circle would intersect at least one of the coordinate axes, contradicting the given condition, so the center must lie in the first quadrant. Let the coordinates of the center be (a,a) ; then the radius of the circle is a , and the standard equation of the circle is $(x-a)^2+(y-a)^2=a^2$. By the given condition, $(2-a)^2+(1-a)^2=a^2$, which gives $a^2-6a+5=0$, solving to $a=1$ or $a=5$, so the coordinates of the center are $(1,1)$ or $(5,5)$, and in both cases the distance from the center to the line $2x-y-3=0$ is $d=\frac{|-2|}{\sqrt{5}}=\frac{2\sqrt{5}}{5}$. Therefore the distance from the center to the line $2x-y-3=0$ is $\frac{2\sqrt{5}}{5}$. Hence the answer is B.

41. A From the given, $\begin{cases} S_4=4a_1+\frac{d}{2}\times 4\times 3=0 \\ a_5=a_1+4d=5 \end{cases}$, solving gives $\begin{cases} a_1=-3 \\ d=2 \end{cases}$, $\therefore a_n=2n-5$. Hence A.

42. D Since $\cos\alpha=1-2\sin^2\frac{\alpha}{2}=\frac{1+\sqrt{5}}{4}$ and α is an acute angle, solving gives $\sin\frac{\alpha}{2}=\sqrt{\frac{3-\sqrt{5}}{8}}=\sqrt{\frac{(\sqrt{5}-1)^2}{16}}=\frac{\sqrt{5}-1}{4}$. Hence the answer is D.

43. C Let the equilateral hyperbola be $c: x^2-y^2=a^2(a>0)$. The directrix of $y^2=16x$ is l: $x=-4$. $\because$ C intersects the directrix $1: \mathrm{x}=-4$ of the parabola $\mathrm{y}^2=16\mathrm{x}$ at points A,B , with $|\mathrm{AB}|=4\sqrt{3}$, $\therefore$ $A(-4,2\sqrt{3}), \mathrm{B}(-4,-2\sqrt{3})$. Substituting the coordinates of point A into the hyperbola equation gives $a^2=(-4)^2-(2\sqrt{3})^2=4$, $\therefore a=2, \quad 2a=4$. The answer is C.

44. A In $\triangle ABC$, AD is the median on side BC and E is the midpoint of AD , so $\overrightarrow{\mathrm{EB}}=\overrightarrow{\mathrm{AB}}-\overrightarrow{\mathrm{AE}}=\overrightarrow{\mathrm{AB}}-\frac{1}{2}\overrightarrow{\mathrm{AD}}$ $=\overrightarrow{\mathrm{AB}}-\frac{1}{2}\times\frac{1}{2}(\overrightarrow{\mathrm{AB}}+\overrightarrow{\mathrm{AC}})=\frac{3}{4}\overrightarrow{\mathrm{AB}}-\frac{1}{4}\overrightarrow{\mathrm{AC}}$. So the answer is A.

45. B $\because S_n$ is the sum of the first n terms of the arithmetic sequence $\{a_n\}$, and $3S_3=S_2+S_4, a_1=2$, $\therefore 3\times\left(3\mathrm{a}_1+\frac{3\times 2}{2}\mathrm{d}\right)=\mathrm{a}_1+\mathrm{a}_1+d+4\mathrm{a}_1+\frac{4\times 3}{2}\mathrm{d}$, substituting $a_1=2$ gives $d=-3$, $\therefore a_5=2+4\times(-3)=-10$. The answer is B.

46. B Since $f(x)$ is an even function, $f(1)=f(-1)\measuredangle:(1+a)\ln\frac{1}{3}=(-1+a)\ln 3$, solving gives $a=0$. When $a=0$, $f(x)=x\ln\frac{2x-1}{2x+1}$, $(2x-1)(2x+1)>0$, which gives $x>\frac{1}{2}$ or $x<-\frac{1}{2}$, so the domain is $\{x|x\rangle\frac{1}{2}$ or $x<-\frac{1}{2}\}$, which is symmetric about the origin.

$$f(-x)=(-x)\ln\frac{2(-x)-1}{2(-x)+1}=(-x)\ln\frac{2x+1}{2x-1}=(-x)\ln\left(\frac{2x-1}{2x+1}\right)^{-1}=x\ln\frac{2x-1}{2x+1}=f(x),$$

so in this case $f(x)$ is an even function. Hence the answer is B.

47. C For A, $y=x^2+2x+4=x^2+2x+1+3=(x+1)^2+3\geq 3$, which does not meet the requirement. For B, $y=|\sin x|+\frac{4}{|\sin x|}$: let $t=|\sin x|\in[0,1]$, $\quad\therefore y=t+\frac{4}{t}$; by the properties of this function, $y_{\min}=1+4=5$, which does not meet the requirement. For C, $y=2^x+2^{2-x}=2^x+\frac{4}{2^x}$: let $t=2^x>0$, $\therefore y=t+\frac{4}{t}\geq 2\sqrt{t\cdot\frac{4}{t}}=2\times 2=4$ with equality if and only if $t=2$, which meets the requirement. For D, $y=\ln x+\frac{4}{\ln x}$: let $t=\ln x\in R$, $\quad y=t+\frac{4}{t}$; by the properties of this function, $y\in(-\infty,-4]\cup[4,+\infty)$, which does not meet the requirement. The answer is C.

48. D The function $y=2^x$ is monotonically increasing on $\mathbf{R}$, and $f(x)=2^{x(x-a)}$ is monotonically decreasing on the interval $(0,1)$, so the function $y=x(x-a)=\left(x-\frac{a}{2}\right)^2-\frac{a^2}{4}$ must be monotonically decreasing on the interval $(0,1)$. Therefore $\frac{a}{2}\geq 1$, which gives $a\geq 2$, so the range of a is $[2,+\infty)$. The answer is D.